MATHEMATICS IN THE PROFESSIONAL SCHOOL. II.

PEDAGOGY OF ELEMENTARY MATHEMATICS.

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BLINDED to educational values by zeal for their subject, or biased by the desire to go the easier way, many teachers of elementary mathematics still argue, as almost everybody argued fifty years ago, in favor of the high disciplinary value of the pure memory work of arithmetic. Notwithstanding the fact that fifty years' experiences are against this view and that psychologists are pretty well agreed that memory is the one mental faculty that cannot be cultivated, some of us still cling to the traditionary notion. That a more discriminating insistence, by friends of mathematics, upon the relative educational values of the various aspects of arithmetical study would redound to the credit of this study as well as to the benefit of education, there can be little room for doubt. Drill work, *memoriter* exercises, committing rules, and the other things which make up the *mechanics* of arithmetic, have no great educational value in themselves, and very much of such work is stupefying. On the contrary, analyses of the conditions on which the solution of problems depends involve the exercise of attention, comparison, and judgment, and have therefore a high value in education. These statements are corroborated by the circumstance that very few great mathematicians, either as boys or as men, ever manifested more than ordinary skill, and many of them not even ordinary skill, in the purely mechanical parts of mathematical study.

The mechanical work of arithmetic should be reduced to the automatic as soon as possible. All teachers have observed that the mechanical parts of arithmetic are reduced to the automatic state much sooner by some pupils than by others. It does not seem to be a matter of common observation, however, that the great mathematicians come from the *dilatory* class. On closer

examination we readily recognize in this the truth that a quick and retentive memory conduces to superficiality. As elementary mathematics is commonly taught, memory is too often the only faculty appealed to, so that the paradoxical rule may usually be followed with safety: "Pick the future mathematician from among those pupils who do not get on well in their elementary mathematics."

But this need not be so. It would not be so if in our teaching of arithmetic we should pay more attention to the needs of the growing faculties of the child than to the ill-considered assertions of bank officials as to the overmastering importance of accuracy and speed in the mechanics of arithmetic. Premature accuracy is one of the worst things the mathematical teacher can strive for. On this point it would be well for the teacher to bear in mind that the bank official's ideal employee is a *machine*, not a *man*. Let us therefore look to a higher source than the mart for our mathematical pedagogy!

There has been not a little theorizing of late on the possibility of fixing the number facts of addition and multiplication upon the memory with sufficient permanence through *use*. Believing, though the writer does, in this *possibility*, he has not as yet been privileged to see this possibility actualized. Still he is encouraged not to discard his belief by the consideration that he has not yet seen a conscientious and consistent effort to realize this possibility put forth by a teacher whose grasp of mathematical truth was any augury of success. Such an effort by such a teacher would, however, go far toward giving an educative value to the part of mathematical study which is a burden to teachers and a bugbear to pupils.

Finally, we shall be better able to understand why this subject of arithmetic, which has never had a large number of friends among educators, still holds so important a place in the curriculum, if we recognize that the demand for the subject is social and industrial rather than educational. We could not discard it if we would, since it is the *sine qua non* of success in so many other fields of human activity. The work of the class-room should therefore be based largely on such problems

as are met in everyday life, and the right sort of mathematical pedagogy will render proper account of these *extra*-curricular demands. The outlines which follow are a continuation of those in the October number of the ELEMENTARY SCHOOL TEACHER:

B. NOVEMBER OUTLINE.

I. Further use of linear, square, and cubical measure and weights applied to concrete and useful problems involving the four fundamental processes. Problems to be taken largely from the other subjects.

II. Notation and numeration of large numbers (to 10,000).

III. Short and long division and simple fractions.

IV. Decimal fractions begun in connection with problems (*a*) in United States money and (*b*) in the units of metric system.

V. Multiplication table and tables of denominate numbers mastered and used continuously.

VI. Extensive applications of fundamental processes in problems of daily life.

VII. Beginnings of geometrical surveying.

VIII. Use of simple equations and their transformations.

IX. Common and decimal fractions completed.

X. Foundations of percentage laid.

XI. Simple scale drawing and graphical representations.

XII. Making drawings for use in manual training.

XIII. Making apparatus in manual training from drawings.

XIV. Meteorological records discussed and reduced.

C. DECEMBER OUTLINE.

I. Arabic, Roman, and index notations and numerations mastered.

II. Use of checks and short methods.

III. Mensuration of common surfaces and solids.

IV. Symbolic representation of arithmetical laws.

V. Practical problems involving more extended analyses and computations.

VI. Indirect measurement—land surveying.

VII. Observational astronomy.

VIII. Problems relating to business, banking, discount, etc.

IX. Graphical representations.

X. Solution of some problems both algebraically and arithmetically.

XI. Positive and negative numbers used.

XII. Practical geometrical constructions.

D. OUTLINE FOR ELEMENTARY MATHEMATICAL PEDAGOGY (WINTER QUARTER).

The course for the winter quarter may be outlined under the following captions:

I. Aims that have hitherto controlled in the teaching of arithmetic: (*a*) practical, (*b*) utilitarian, (*c*) theoretical.

II. Why have these aims controlled? Contemporaneous sociological and civic demands.

III. Modern aims.

IV. Origin and nature of the number concept: (*a*) the objective element (multiplicity of things); (*b*) the subjective element (relating faculty); (*c*) numbering is *ordering*, *vs.* numbering is *measuring*.

V. The most important aspects of number used by text-book writers: (*a*) group aspect; (*b*) measuring aspect; (*c*) ratio aspect.

VI. Methods, so called, of teaching numbers: (*a*) method of symbols; (*b*) of things; (*c*) logical method; (*d*) psychological; (*e*) ratio method; (*f*) fixed-unit method; (*g*) variable-unit method; (*h*) spiral method.

VII. Function of number most used in: (*a*) daily life; (*b*) science; (*c*) education; (*d*) number theory.

VIII. Physical and mental needs which give rise to the operations of: (*a*) addition and subtraction; (*b*) multiplication and division; (*c*) fractions and percentage; (*d*) involution and evolution.

IX. Reasons for and against introducing elements of algebra and geometry in grades.

X. Changes and improvements needed in the teaching of elementary mathematics: (*a*) explicit use of the equation in arithmetic; (*b*) some topics dropped, some introduced; (*c*) arithmetic should be given a sociological aspect; (*d*) improvements in language of science; (*e*) better preparation of teachers.

XI. Number-work of nature-study, history, geography, manual training: (*a*) virtues; (*b*) vices.

XII. Introduction of work from industrial occupations.

XIII. Values of arithmetic in education: (*a*) practical value; (*b*) culture value.

E. OUTLINE FOR SPRING QUARTER, ON MATHEMATICAL PEDAGOGY FOR THE SECONDARY SCHOOL.

I. Mathematical subjects in the secondary school.

II. The bases of geometry, algebra, and trigonometry: (*a*) axioms; (*b*) definitions.

III. General methods of establishing truth in geometry: (1) equality by superposition; (2) mode of proving inequality; (3) principle of *reductio ad absurdum;* (4) principle of parallelism; (5) principles for measurement of central and inscribed angles; (6) principle of reducing the mensuration of all plane areas to that of the triangle; (7) principle of limits; (8) principle of reducing solid geometry to plane by analysis of figures.

IV. What mathematical knowledge the pupil should have on entering the secondary school.

V. Needs of students: (*a*) who do not go beyond the high school; (*b*) who expect to attend a university, or technical school.

VI. Simultaneous teaching of algebra and geometry.

VII. Correlation of mathematical subjects with science subjects: (1) from standpoint of pupil; (2) from standpoint of teacher's preparation.

IX. What is the aim of the study of geometry? of algebra?

X. Algebra: (*a*) as a preparation for the study of functions; (*b*) as a study of the equation and laws of its use.

XI. Exemplification of algebraic principles by the numbers of arithmetic.

XII. Use and abuse of axioms in transforming equations.

XIII. Sequence of topics in teaching of: (*a*) geometry; (*b*) algebra.

XIV. Classification of equations with reference to degree; graphical representation of their solutions.

XV. Fundamental principles of factoring.

XVI. Questions to be raised on taking each new step: (*a*) why is it correct? (*b*) can it be reversed?

XVII. Field and laboratory work in algebra and geometry: (*a*) geometrical surveying: (*b*) solution and discussion of the equations of science and engineering.

XVIII. Place and relation of secondary trigonometry.

XIX. Observational astronomy and constellation study.

XX. Correlation of mathematics and physics in the secondary school: (*a*) argument from standpoint of subject-matter; (*b*) argument from standpoint of pupil; (*c*) argument from standpoint of teachers' preparation.

XXI. Study of leading texts in algebra and geometry for the high school.